NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	**LEVEL:** 6
COURSE CODE: SIN601S	**COURSE NAME:** STATISTICAL INFERENCE 2
SESSION: JULY 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr D.B. GEMECHU
MODERATOR:	Dr C.R. KIKAWA

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1 [20 marks]

1. Let $Y_1 < Y_2 < \cdots < Y_5$ be the order statistics of 5 independently and identically distributed continuous random variables $X_1, X_2, \ldots, X_5$ with pdf f given by

$$f(x) = \begin{cases} 2x & for\ 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Then find

1.1. The pdf of the r^{th} order statistics. **Hint:** $f_{Y_r}(y) = nf_X(y)\binom{n-1}{r-1}[F_X(y)]^{r-1}[1 - F_X(y)]^{n-r}$ **[3]**

1.2. The pdf of the minimum order statistics **[3]**

1.3. The pdf of the maximum order statistics **[3]**

1.4. The pdf of the median **[3]**

1.5. The joint pdf of $Y_1, Y_2, \ldots, Y_5$ **[3]**

1.6. The joint pdf of the minimum and maximum order statistics. **[5]**

Hint: $f_{Y_i,Y_j}(y_i, y_j) = \frac{n!}{(i-1)!(j-i-1)!(n-j)!}[F_X(y_i)]^{i-1} f_X(y_i)[F_X(y_j) - F_X(y_i)]^{j-i-1} f_X(y_j)[1 - F_X(y_j)]^{n-j}$

QUESTION 2 [8 marks]

2. Let $Z_1, Z_2, \ldots . Z_n$ be independently and identically distributed with standard normal distribution having $E(Z_i) = 0$ and $V(Z_i) = 1$. Then show, using the moment generating function, that $Y = \sum_{i=1}^{n} Z_i$ has a normal distribution and what will be the mean and variance of Y? (**Hint:** If $X \sim N(\mu, \sigma^2)$, then $M_X(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$). **[8]**

QUESTION 3 [9 marks]

3. Let X_1, X_2, X_3, X_4 be a random sample from a distribution with density function

$$f(x_i|\beta) = \begin{cases} \frac{1}{\beta} e^{-\frac{(x_i - 4)}{\beta}} & for\ x_i > 4 \\ 0 & otherwise \end{cases}$$

where $\beta > 0$.

3.1. Find the maximum likelihood estimator of β **[6]**

3.2. If the data from this random sample are 8.2, 9.1, 10.6 and 4.9, respectively, what is the maximum likelihood estimate of β? **[3]**

QUESTION 4 [6 marks]

4. Let $Y_1, Y_2, \ldots, Y_n$ be n independent random variables such that each $Y_i \sim Poisson(\beta x_i)$, where β is an unknown parameter. If $\{(y_1, x_1), (y_2, x_2), \ldots, (y_n, x_n)\}$ is a dataset where $y_1, y_2, \ldots, y_n$ are the observed values based on $x_1, x_2, \ldots, x_n$, then find the least estimator of β. **Hint:** If $Y \sim Poisson(\theta)$, then $E(Y) = \theta$ **[6]**

QUESTION 5 [14 marks]

5. Let $X_1,\ X_2,\ X_3,\ and\ X_4$ be a random sample of observations from a population with mean mean μ and variance σ^2. Consider the following two point estimators of μ

$$\hat{\theta}_1 = 0.10X_1 + 0.20X_2 + 0.40X_3 + 0.30X_4 \text{ and}$$

$$\hat{\theta}_2 = 0.25X_1 + 0.25X_2 + 0.30X_3 + 0.20X_4$$

5.1. Show that both estimators are unbiased estimators of μ. **[4]**

5.2. Find the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$. **[8]**

5.3. Which estimator is more efficient, $\hat{\theta}_1$or $\hat{\theta}_2$? Explain in detail. **[2]**

QUESTION 6 [27 marks]

7. Let $X_1, X_2, \ldots, X_n$ be a random sample from the exponential distribution with the parameter θ and the probability density function x_i is given by

$$f(x_i|\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{1}{\theta}x_i} & for\ x_i > 0 \\ 0 & otherwise \end{cases}$$

7.1. Show that the mean and variance of X_i are θ and θ^2 respectively. **[8]**

Hint: $M_{X_i}(t) = \left(\frac{1}{1-\theta t}\right)$

7.2. Show that $\bar{X}$ is a sufficient statistics for the parameter θ. **[6]**

7.3. Show that $\bar{X}$ is a minimum variance unbiased estimator (MVUE) of θ. **[13]**

QUESTION 7 [16 marks]

9. Suppose one observation was taken of a random variable X which yielded the value 3. The density function for X is

$$f(x|\theta) = \begin{cases} \theta^{-1} & for\ 0 < x < \theta \\ 0 & otherwise \end{cases}$$

and the prior distribution of θ is

$$h(\theta) = \begin{cases} 2\theta^{-2} & for\ 1 < \theta < \infty \\ 0 & otherwise \end{cases}$$

9.1. Find the posterior distribution of θ. **[7]**

9.2. If the squared error loss function is used, find the Bayes' estimate of θ. **[4]**

9.3. If an absolute difference error loss function is used, find the Bayes' estimate of θ. **[5]**

=== END OF PAPER===

TOTAL MARKS: 100